

# Vacuum Polarization Contribution to the Astrophysical S-Factor

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## Abstract

We study the effect of vacuum polarization in nuclear reactions of astrophysical interest. This effect has the opposite sign compared to the screening by the atomic electrons. It is shown that vacuum polarization further increases the longstanding differences between the experimental data of astrophysical nuclear reactions at very low energies and the theoretical calculations which aim to include electron screening, albeit by a small amount.

Understanding the dynamics of fusion reactions at very low energies is essential to understand the nature of stellar nucleosynthesis. These reactions are measured at laboratory energies and are then extrapolated to thermal energies. This extrapolation is usually done by introducing the astrophysical S-factor:

$$\sigma(E) = \frac{1}{E} S(E) \exp(-2\pi\eta) , \quad (0.1)$$

where the Sommerfeld parameter,  $\eta$ , is given by  $\eta = Z_1 Z_2 e^2 / \hbar v$ . Here  $Z_1$ ,  $Z_2$ , and  $v$  are the electric charges and the relative velocity of the target and projectile combination.

The term  $\exp(-2\pi\eta)$  is introduced to separate the exponential fall-off of the cross-section due to the Coulomb interaction from the contributions of the nuclear force. The latter is represented by the astrophysical S-factor which is expected to have a very weak energy dependence. The form given in Eq. (1) assumes that the electric charges on nuclei are “bare”. However, neither at very low laboratory energies, nor in stellar environments this is the case. In stars the bare Coulomb interaction between the nuclei is screened by the electrons in the plasma surrounding them. A simple analytic treatment of plasma screening was originally given by Salpeter [1]. In most cases of astrophysical interest Salpeter’s treatment still remains to be a sufficient approximation [2]. In the very low energy laboratory experiments the bound electrons in the projectile or the target may also screen the Coulomb potential as the outer turning point gets very large ( $> 500$  fm). As experimental techniques improve one can measure the cross section in increasingly lower energies where the screened Coulomb potential can be significantly less than the bare one. This deviation from the bare Coulomb potential should manifest itself as an increase in the astrophysical S-factor extracted at the lowest energies. This enhancement was indeed experimentally observed for a large number of systems [4–8]. The screening effects of the atomic electrons can be calculated [3] in the adiabatic approximation at the lowest energies and in the sudden approximation at higher energies with a smooth transition in between [9]. Contributions from the nuclear recoil caused by the atomic electrons are expected to further increase the screening effect for asymmetric systems [9,10]. In almost all the cases observed screening effects are found to be equal to or more than the theoretical predictions. Recently including improved energy loss data for atomic targets is shown to lead agreement between theory and data [11], however the situation is still not resolved for molecular and solid targets. Electron screening enhancement was not observed for the heavier symmetric system  ${}^3\text{He}({}^3\text{He}, 2p){}^4\text{He}$  [12] which is expected to have about 20% enhancement at the energies studied. Recent measurements [13] have not yet clarified the effects of electron screening in this reaction. A mechanism which reduces the screening enhancement for this system (and possibly for other systems

with large values of  $Z_1 Z_2$  and the reduced mass) seems to be needed. In this note we show that the contributions from the polarization of the vacuum alone cannot achieve this task. The effects of the polarizability was previously investigated in Ref. [14] for elastic scattering below the Coulomb barrier and in Ref. [15] for subbarrier fusion reactions using the formalism developed by Uehling [16]. Effects of vacuum polarization in  $^{12}\text{C} - ^{12}\text{C}$  scattering at 4 MeV was subsequently experimentally observed [17].

Vacuum polarization **increases** the electromagnetic potential between two like charges. Like the Coulomb potential itself, the increase due to vacuum polarization is also proportional to the product of the charges [16]. Vacuum polarization contribution increases almost exponentially as the two charges get closer. The Coulomb interaction is smaller for asymmetric systems than for symmetric systems of comparable size. On the other hand, the nuclear force tends to extend farther out for asymmetric systems because of the extra neutrons. Consequently for asymmetric systems the very tail of the nuclear force can turn the relatively weak Coulomb potential around to form a barrier at a considerable distance from the nuclear touching radius. For symmetric systems, however, the location of the barrier is further inside where the vacuum polarization contribution is stronger. We show that the resulting increase in vacuum polarization is nevertheless not sufficiently large to make an appreciable contribution to the extracted astrophysical S-factor. For light symmetric systems with small values of  $Z_1 Z_2$  this effect should be negligible. Indeed, for the  $pp$  reaction the vacuum polarization contribution was shown to be very small [18]. Similarly the measured S-factor for the  $d(d, p)^3\text{H}$  reaction [8] agrees well with theoretical calculations of atomic screening [19]. On the other hand one may expect that already for the  $^3\text{He}(^3\text{He}, 2p)^4\text{He}$  reaction the increase in the potential due to the vacuum polarization could be large enough to counter the decrease due to electron screening. We show that this is not the case.

The vacuum polarization potential is according to Uehling [16] given by

$$V_{Pol}(r) = \frac{Z_1 Z_2 e^2}{r} \frac{2\alpha}{3\pi} I\left(\frac{2r}{\lambda_e}\right),$$

where  $\alpha = 1/137$  is the fine structure constant, and  $\lambda_e = 386$  fm is the Compton wavelength of the electron. The function  $I(x)$  is given by

$$I(x) = \int_1^\infty e^{-xt} \left(1 + \frac{1}{2t^2}\right) \frac{\sqrt{t^2 - 1}}{t^2} dt.$$

As shown by Pauli and Rose [20] this integral can be rewritten as

$$I(x) = \alpha(x)K_0(x) + \beta(x)K_1(x) + \gamma(x) \int_x^\infty K_0(t) dt,$$

where

$$\begin{aligned}\alpha(x) &= 1 + \frac{1}{12}x^2, & \beta(x) &= -\frac{5}{6}x(1 + \frac{1}{10}x^2), \\ \gamma(x) &= \frac{3}{4}x(1 + \frac{1}{9}x^2), & \text{with } x &= 2r/\lambda_e.\end{aligned}$$

In Ref. [14] it was shown that the modified Bessel functions  $K_0$  and  $K_1$  as well the integral over  $K_0$  can be expanded in a very useful series in Chebyshev polynomials which converge rapidly and for practical purposes only a few terms ( $\approx 5 - 10$ ) is needed, allowing a very fast and accurate computation of the Uehling potential.

In Figure 1 we plot the Coulomb potential and the vacuum polarization potential for  $Z_1Z_2 = 1$ . Both the Coulomb potential and the screening potential scale with the product  $Z_1Z_2$ . However, the vacuum polarization potential has a stronger dependence on the nuclear separation distance.

To calculate the effects of screening by the atomic electrons, and the effect of vacuum polarization we use for simplicity the (s-wave) WKB penetrability factor

$$P_{C+Scr+Vac.Pol.}(E) = \exp\left\{-\frac{2}{\hbar} \int_{R_n}^{R_C} dr [2m(V_c + V_{pol} - E')]^{1/2}\right\}, \quad (0.2)$$

where  $E' = E + U_e$ , with  $E$  being the relative energy between the nuclei, and the atomic screening correction, assumed to be a constant, is given by  $U_e$ . The limits of the integral are the nuclear radius,  $R_n$ , where the nuclear fusion reaction occurs, and the classical turning point in the Coulomb potential,  $R_C = Z_1Z_2e^2/E'$ . At very low energies the inferior limit  $R_n$  is not important when vacuum polarization is neglected (the exponential factor in Eq. 1 can be obtained with  $V_{pol} = 0$ , and  $R_n \rightarrow 0$ , in Eq. 2). However, since the vacuum polarization potential has a strong dependence on the nuclear separation distance, being much stronger at shorter distances, its effect is very much dependent on the choice of this parameter. For the sake of simplicity, we use the conventional approximation  $R_n = 1.2 (A_1^{1/3} + A_2^{1/3})$  fm for the nuclear radius. In Table 1 we show the ratio between the penetrability factor through the Coulomb barrier and the penetrability factor including atomic screening,  $P_{C+Scr}(E)/P_C(E)$ , and also including the effect of vacuum polarization,  $P_{C+Scr+Vac.Pol.}(E)/P_C(E)$ . The energy  $E$  chosen is the lowest experimental energy for each reaction. The atomic screening corrections  $U_e$  were calculated in the adiabatic approximation, given by the differences in electron binding energies between the separated atoms and the compound atom [9]. We see that the effect of vacuum polarization is small, but non-negligible for some reactions. Moreover, it increases the discrepancy between the value of the screening potential required to explain the experimental data and the theoretical calculations of this potential as illustrated in Table 1.

In conclusion, we showed that the vacuum polarization contribution to the astrophysical S-factor never exceeds a few percent, but may be significant in extrapolating the measured S-factor to lower energies. Although its contribution is not comparable to that of sub-threshold resonances and electron screening, vacuum polarization is one of the many factors that may contribute to the weak energy dependence of the S-factor. Vacuum polarization effects are sensitive to the inner turning point of the potential barrier, hence to the diffuseness of the nuclear potential employed.

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Reaction	$E_{\min}[keV]$	$U_e[eV]$	$P_{C+Scr}/P_C$	$P_{C+Scr+Vac.Pol.}/P_C$
$D(d, p)T$	1.62	20	1.163	1.150
${}^3He(d, p){}^4He$	5.88	119	1.331	1.303
$D({}^3He, p){}^4He$	5.38	65	1.195	1.173
${}^3He({}^3He, 2p){}^4He$	25	292	1.196	1.159
${}^6Li(p, \alpha){}^3He$	10.74	186	1.258	1.231
${}^7Li(p, \alpha){}^4He$	12.70	186	1.197	1.173
${}^6Li(d, \alpha){}^4He$	14.31	186	1.218	1.186
$H({}^6Li, \alpha){}^3He$	10.94	186	1.250	1.224
$H({}^7Li, \alpha){}^4He$	12.97	186	1.191	1.167
$D({}^6Li, \alpha){}^4He$	15.89	186	1.183	1.153
${}^{10}B(p, \alpha){}^7Be$	18.70	346	1.376	1.338
${}^{11}B(p, \alpha){}^8Be$	16.70	346	1.461	1.419

**Table Caption:**

Lowest experimental energies,  $E_{\min}$ , energy corrections [19] due to the screening by the atomic electrons,  $U_e$ , and enhancement factors for the nuclear reaction: (a) due to atomic screening,  $P_{C+Scr}/P_C$ , and (b) due to the combined effect of atomic screening and vacuum polarization,  $P_{C+Scr+Vac.Pol.}/P_C$ .

**Figure Caption**

**Fig. 1** - Comparison between the Coulomb potential and the vacuum polarization potential as a function of the nuclear separation distance for  $Z_1 Z_2 = 1$ . The vacuum polarization potential has been multiplied by a factor 1000 in order to be visible in the same plot.